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CERTAIN STATICS-RELATED PROBLEMS OF CIRCULAR ORTHOTROPIC AND ISOTROPIC PLATES

In the paper, the author considers a relevant problem of structural mechanics. The antisymmetric bending of constant thickness orthotropic and isotropic circular plates, resting on the elastic Winkler foundation, is the subject of the research. Supplemental analytical solutions are obtained. Solutions are represented as Bessel functions.

Problems of symmetric and asymmetric flexure of isotropic circular plates, resting on the Winkler foundation, enjoy extensive coverage in the literature [1].

In the above-mentioned cases, the solution of the differential equation of the fourth order with variable coefficients decomposes into two differential equations of the second order. This procedure fails in case of orthotropic plates. The Nielsen equation is used to resolve the above problem:

$$r^4 \frac{d^4w}{dr^4} + A_3 r^3 \frac{d^3w}{dr^3} - A_2 r^2 \frac{d^2w}{dr^2} + A_1 r \frac{dw}{dr} + A_0 w = 0,$$

where

$$A_3 = 6 - 4a - 4c,$$

$$A_2 = 2\left(a^2 - \mu^2 c^2\right) + 4\left(a + c - 1\right)^2 + 4\left(a - 1\right)\left(c - 1\right) - 1,$$

$$A_1 = \left[2\left(\mu^2 c^2 - a^2\right) - \left(2a - 1\right)\left(2c - 1\right)\right]\left(2a + 2c - 1\right),$$

$$A_0 = \left(a^2 - \mu^2 c^2\right)\left(a^2 + 4ac + 4c^2 - \mu^2 c^2\right) - b^4 c^4 r^{4c}.$$

The general solution of the homogeneous equation is represented as:

$$w = r^a \left[C_1 J_\mu(u) + C_2 Y_\mu(u) + C_3 I_\mu(u) + C_4 K_\mu(u)\right],$$

where $u = br^c$; J_μ, Y_μ, I_μ и K_μ — the Bessel functions (cylindrical functions of the first and the second kind, modified Bessel and McDonald functions).

The paper represents an essential generalization of the research of professor Conway obtained for the case of the axially symmetric flexure of an isotropic circular plate, resting on the Winkler foundation.

Currently, numerous software programmes designated for the analysis of buildings and structures are available. In these programs, numerical methods, namely, the finite element method, are used. The exact results presented in this paper can be used to assess the accuracy of numerical results.

Key words: orthotropic plates, elastic foundation, antisymmetric distortion, Bessel functions.

References

1. Korenev B.G. *Vvedenie v teoriyu besselevykh funktsiy* [Introduction into the Theory of Bessel Functions]. Moscow, Nauka Publ., 1971, 288 p.

2. Koreneva E.B. *Analiticheskie metody rascheta plastin peremennoy tolshchiny i ikh prakticheskie prilozheniya* [Analytical Methods of Analysis of Plates of Variable Thickness and Their Practical Applications]. Moscow, ASV Publ., 2009, 238 p.

3. Kamke E. *Spravochnik po obyknovennym differentsial'nym uravneniyam* [Reference Book of Ordinary Differential Equations]. Moscow, Nauka Publ., 1965, 703 p.

4. Koreneva E.B., Grosman V.R. Nekotorye voprosy rascheta ortotropnykh plastin, lezhashchikh na uprugom osnovanii, i issledovaniya osesimmetrichnykh kolebaniy kruglykh ortotropnykh plastin [Certain Problems of Analysis of Orthotropic Plates, Resting on the Elastic Foundation, Research of Axis-Symmetric Vibrations of Circular Orthotropic Plates]. Collection of works no. 14, part 1 «Problems of Applied Mathematics and Computing Mechanics». Moscow, MSUCE, 2011, pp. 176—178.

5. Koreneva E.B. Grosman V.R. Analiticheskoe reshenie zadachi ob izgibe krugloy ortotropnoy plastiny peremennoy tolshchiny, lezhashchey na uprugom osnovanii [Analytical Solution of the Problem of Bending of Variable Thickness Circular Orthotropic Plates, Resting on the Elastic Foundation]. *Vestnik MGSU* [Proceedings of Moscow State University of Civil Engineering]. 2011, no. 8, pp. 156—159.

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